Security-Efficiency Tradeoffs in Searchable Encryption

Lower Bounds and Optimal Constructions

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Searchable Encryption

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- Perform search operations ...
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- Allow some leakage to improve performance.

Security *vs.* Efficiency

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Nothing comes for free. Ever!

Efficiency

Many possible measurements:

- Computational complexity
- Communication complexity
- Number of interactions
- Size of the encrypted database
- Size of the client's state
- Memory locality & read efficiency

Security

We can evaluate the security

- formally: from the leakage in the security proofs
- practically: from actual attacks (*e.g.* leakage-abuse attacks)

This presentation

Lower bounds on the efficiency of:

- static searchable encryption schemes hiding the repetition of search queries;
- dynamic searchable encryption schemes with forward-private updates.

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- single-keyword search queries
- database structure: atomic keyword/document pairs (a.k.a. entries)

Notations

- $N = |\mathrm{DB}|$: total number of entries
- K: number of distinct keywords
- $|\mathrm{DB}(w)| = n_w$: number of entries matching w
- a_w: number of entries matching w inserted in the database
- σ: size of the client's state
- $H = (\mathrm{DB}, r_1, \ldots, r_i)$: query history ($r_i$ can be a search query, or an update query)

Security model

- Indistinguishability-based security definition: two executions with the same leakage cannot be distinguished by an adversary
- Only the non-adaptive version of the definition is needed here

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$H^0 = (\mathrm{DB}^0, r_1^0, \ldots, r_i^0)$ $H^1 = (\mathrm{DB}^1, r_1^1, \ldots, r_i^1)$

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$\Downarrow$ Leakage $\Downarrow$

$$\mathcal{L}(H^0) = \mathcal{L}(H^1)$$

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$$\Downarrow \quad \text{Execute} \quad \Downarrow$$

$$\tau^0 \quad \approx \quad \tau^1$$

Schemes hiding the search pattern

- Static schemes only revealing the number of results of a query (hides the repetition of queries)

$$\mathcal{L}(\mathsf{DB}, w_1, \ldots, w_i) = ((N, K), n_{w_1}, \ldots, n_{w_n})$$

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- Related to ORAM (# results of each query is 1)
  Called File-ORAM in [ACN+17]
- ORAM lower bound [GO96]: $\Omega\left(\frac{\log N}{\log \sigma}\right)$

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- The order of the returned elements does not matter

$$\Omega\left(\frac{\log \binom{N}{n_w}}{\log \sigma}\right)$$

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- As $w \neq w'$, the adversary knows that the accessed entries will be different. The number of entries to consider is

$$\overline{N}(H, w) = N - \sum_{n \in \{|\mathrm{DB}(w_j)| \neq |\mathrm{DB}(w)|\}} n.$$

Lower bound on search-pattern-hiding SSE

Theorem

Let Σ be a static SSE scheme leaking (N, K) and $|\mathrm{DB}(w)|$. Then the complexity of the search protocol is

$$\Omega\left(\frac{\log\binom{\overline{N}(H,w)}{n_w}}{\log|\sigma|\cdot\log\log\binom{\overline{N}(H,w)}{n_w}}\right)$$

where

$$\overline{N}(H,w) = |\mathrm{DB}| - \sum_{n\in\{|\mathrm{DB}(w_j)|\neq|\mathrm{DB}(w)|\}} n.$$

Tightness of the lower bound

w_0	2
w_1	3
w_2	1
w_3	56
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$\vdots$	$\vdots$

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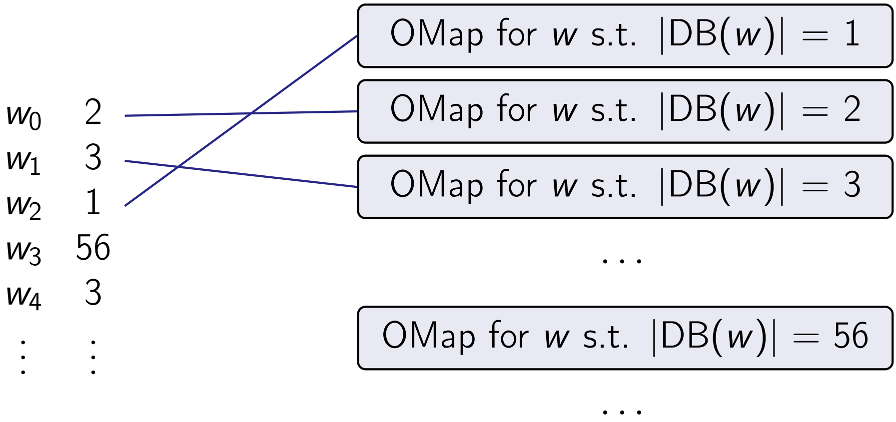

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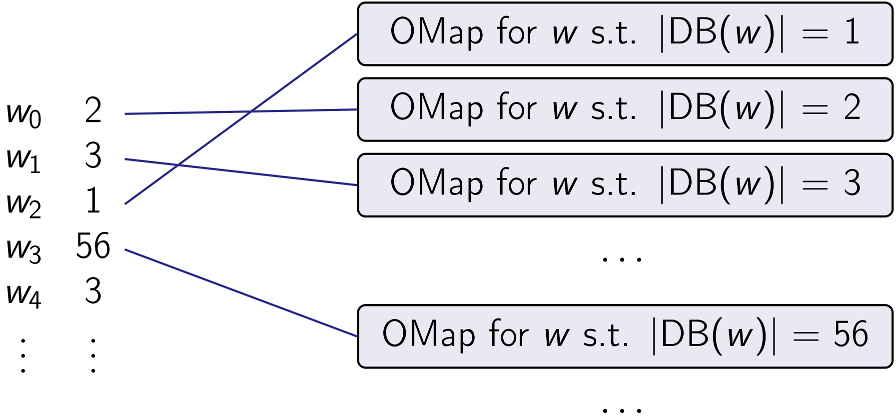

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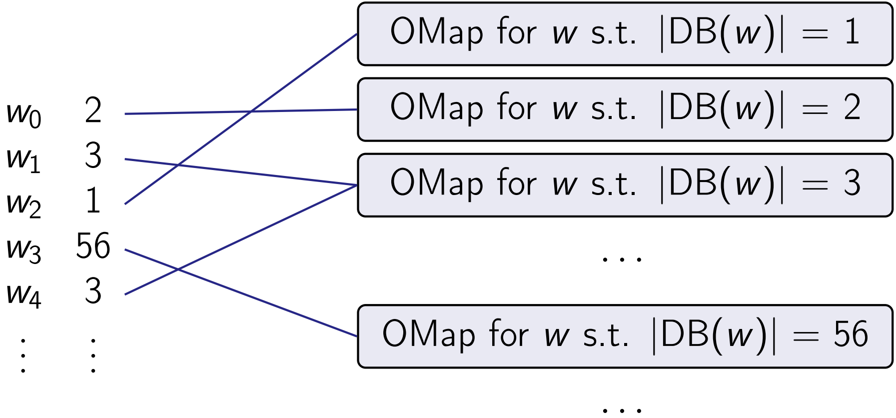

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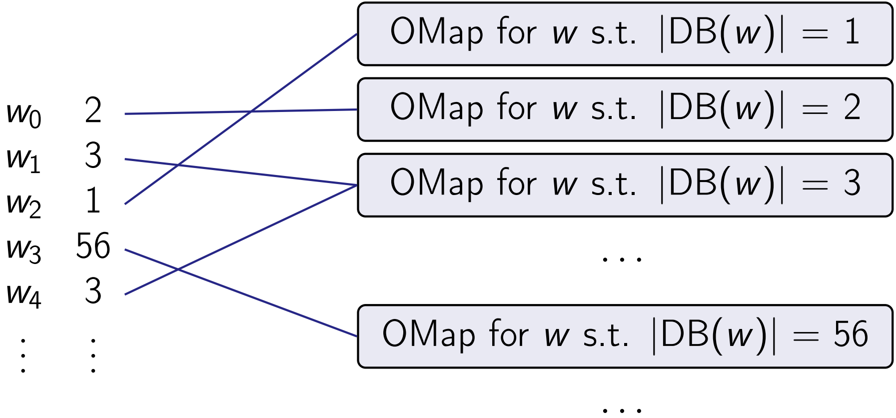

Query complexity of an OMap of size n: $\mathcal{O}\left(\log^2 n\right)$.
The search complexity of the construction is $\mathcal{O}\left(\log^2 K\right)$.

Tightness of the lower bound

When $K \ll N$, the previous construction breaks the lower bound.

During setup, the *profile* of the database is leaked: $(K_i)_{i=1}$ where $K_i = \#\{w \text{ s.t. } |\mathrm{DB}(w)| = i\}$.

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With a small additional leakage, we can break the lower bound on SP-hiding SSE.

Forward Privacy

File injection attacks [ZKP16]

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Introduced in [SPS14], must have security feature for modern dynamic schemes

The cost of forward privacy

Scheme	Computation: Search	Computation: Update	Client Storage	FP	
[CJJ+14]	$\mathcal{O}(a_w)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	✗	
[SPS14]	$\mathcal{O}(a_w + \log N)$ $\mathcal{O}\left(n_w \log^3 N\right)$	$\mathcal{O}\left(\log^2 N\right)$	$\mathcal{O}(N^\alpha)$	✓	Supports deletions well
$\Sigma o\phi o\varsigma$	$\mathcal{O}(a_w)$	$\mathcal{O}(1)$	$\mathcal{O}(K)$	✓	TDP
[EKPE18]	$\mathcal{O}(a_w)$	$\mathcal{O}(1)$	$\mathcal{O}(K)$	✓	write during search
[KKL+17]	$\mathcal{O}(a_w)$	$\mathcal{O}(1)$	$\mathcal{O}(K)$	✓	(write during search)
Diana	$\mathcal{O}(a_w)$	$\mathcal{O}(\log a_w)$	$\mathcal{O}(K)$	✓	CPRF
FAST	$\mathcal{O}(a_w)$	$\mathcal{O}(1)$	$\mathcal{O}(K)$	✓	

Lower bound on forward-private SE

Theorem

Let Σ be a forward-private SSE scheme. Then either the update complexity of an update is

$$\Omega\left(\frac{\log K}{\log|\sigma| \cdot \log\log K}\right)$$

or the complexity of a search is

$$\Omega\left(\frac{t\log K}{\log|\sigma| \cdot \log\log K}\right)$$

t is the number of updates since the last search query.

Tightness of the FP lower bound

- $\Sigma o\phi o\varsigma$, KKLPK, EKPE and FAST show that the lower bound is tight ($|\sigma| = K$).
- FAST shows that the lower bounds can be reached relying only on a PRF, without rewriting the DB during the search algorithm to 'cache' the results.
- Outsource the client's counter map using an oblivious map data structure.
  $|\sigma| = \mathcal{O}(1)$, $\mathcal{O}\left(\log^2 K\right)$ search & update complexity.
- Open question: is there a middle point?
  e.g. $|\sigma| = \mathcal{O}\left(\sqrt{K}\right)$ & $\mathcal{O}(1)$ update complexity.

Conclusion

- Two lower bounds showing the tradeoffs between security and efficiency
- These bounds are (essentially) tight

Thank you!

Slides: https://r.bost.fyi/slides/PETS19.pdf

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